shifting center of mass, changes in central rigid body inertias (with/without deployment), etc.[3-5,7,8]

The system referred to in Ref. 1 can be improved by generalizing the plate model to include at least the in-plane degrees of freedom and to include the effect of axial load coupling. Variable cross sectional properties of the appendages could also be allowed for. However, considering the complexities involved for this class of problem, it is likely more useful and relevant to spend the time and effort to first introduce articulation at the roots of the appendages.

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Reply by Authors to K.W. Lips

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IN his remarks on Ref. 1, Lips does not find the formulation general enough and suggests that the analyses by Lips [2, 5]‡, Lips and Modi [3, 4, 7, 8], and Ibrahim and Misra [6] apparently cover the same ground

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§Numbers in brackets refer to references cited by Lips in his Comment.

Lips seems to suggest that the formulation given in Refs. [2-8] can be used to study beam as well as plate type of flexible appendages. This is grossly misleading and oversimplification of the fact. It is like saying that Newton's laws can be used to study motion of a pendulum as well as of the Space Shuttle. Nothing wrong with the statement. By itself, however, it does not provide any insight into the pendulum motion or the Shuttle dynamics. Papers by Lips and Modi [3, 4, 7, 8] have analyzed only beam-type appendages, mostly two, along the local vertical. The statement,

> "For the case considered in Ref. 1, a single additional subroutine for the plate vibrations is all that is required,"

is rather simplistic. Most dynamicists recognize that solution of challenging problems involves much more than development of a subroutine. In one stroke Lips makes the analysis of Ibrahim and Misra [6] redundant although that is the one he refers to for plate-type appendages. In fact, a vast body of current investigations by Bainum and his colleagues,[5,6] study of the SCOLE configuation,[7-9] tether dynamics,[10,11] and many others would instantly become substantially irrelevant by such a line of thinking.

It seems Lips himself has partly answered his own comment when he says: "The configuration is unique in that it includes both beam and plate appendages." What Lips calls "unique" is really a relatively "general" configuration. Lips and Modi dealt only with beam-type appendages. Ibrahim and Misra took an important step forward and considered two plate-type appendages deploying normal to the orbital plane. In reality, complex spacecraft are likely to have both beam- as well as plate-like structural members in a variety of orientations. The formulation of Modi and Ibrahim aims at such general and realistic configurations covering a vast class of space vehicles including the present and next generation of satellites, the Orbiter based construction of structural components, and several dynamical aspects involved in construction of the proposed space station. For example, consider the NASA/Lockheed Solar Array Flight Experiment. This experiment, which was conducted aboard the Orbiter Discovery, involved deployment and retrieval of a solar panel, 31 m in length, to generate 13 kW of additional power. Considering the solar panel as a plate, Modi and Ibrahim's formulation is able to study the complex dynamics.[2-4] However, none of th references cited by Lips have managed that. What makes the formulation relatively general are the following features:

1) It considers the central rigid body, of any arbitrary inertia distribution, negotiating any specified trajectory and undergoing three-dimensional librational motion.

2) It accommodates an arbitrary number of beam- and plate-type flexible appendages deploying independently at arbitrary velocity and acceleration.

3) It accounts for gravitational effects, shifting center of mass, changing rigid body inertia, and off-sets of appendage attachment locations.

4) Modified Eulerian rotations are so chosen as to make the formulation applicable to both gravity gradient and spin stabilized orientations.

Although Lips does not find this formulation general enough, none of the references cited by him account for all of the aspects mentioned above together. In fact, his analysis applied to beams[3-6,8,9] does not even account for changing rigid body inertia due to deploying mass initially stored in the central rigid body. This is not to minimize the importane of the contribution by Lips and Modi. However, it is important to recognize that progress normally occurs in a gradual measure and there is always room for generalization. Incidentally, Lips' observation that the formulation in Ref. 1 is for uniform beams and plates is not correct. Please refer to the strain energy integrals.

Under the heading "Plate, Beam Similarities," Lips says: "No axial twist or in-plane vibrations are accounted for." Let us look at this point closely. So far as the beam is concerned, transverse oscillations in the two orthogonal directions are considered. For the plate-type flexible member, its motion is represented by superposition of the fixed-free and free-free beam modes. Thus it accounts for transverse as well as twist behaviour. Axial foreshortening effect was purposely omitted. It was found that its contribution is relatively negligible in situations encountered in spacecraft dynamics.

In the section "Absence of Axial Load Effects," Lips mentions axial force and gravity gradient effects. The formulation presented in Ref. 1 is applicable to spinning satellites and accounts for gravity gradient. Lips seems to confuse axial loading and axial foreshortening during vibration.

Lips is correct in noticing the difference between the time rate of change of length and the appendage deployment velocity. In the problem encountered by spacecraft appendages, even extending to 100 m, such distinction is of negligible consequence.

To summarize, the formulation is not an "outline of procedure" but a concrete set of equations valid for a large class of spacecraft with arbitrarily-oriented beam- and plate-type deploying flexible members. It is backed by a functioning numerical algorithm. No paper can provide all the details when complex equations often involve more than a hundred terms. Response results were not given in the paper[1] only because of the constraint of space imposed by the Journal. However, such results have been reported elsewhere.[2-4] Incidentally, the response of plate-type appendages can be quite different. Thus Lips' prediction, ".....it is unlikely that the simulated plate response will appear much different than that of the beam," is off the mark. Further generalization of the formulation accounting for arbitrary, topological tree type additions of structural assemblies, as would be the case during the space station construction, is in progress and will be reported soon.

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Comment on "To Pursue or to Evade— That is the Question"

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THE purpose of this Comment on the highly intriguing paper cited[1] is to suggest that, assuming that both combatants prefer survival to simultaneous kill, the "pursuer" should sometimes decide *not* to pursue, thereby diminishing the size of the state region leading to sumultaneous kill.

When Fig. 4g, for example, of Ref. 1 is modified by the considerations underlying Fig. 7 there, it appears as in Fig. 1 of this Comment. SK denotes the region (denoted as "Evader's Choice" in the original Fig. 7) where the "evader" (B or A according as the relative position lies right or left of the "role-reversal" line (L)) has time to turn toward the pursuer and achieve a simultaneous kill. The upper boundaries of the SK region are determined (as in the original Fig. 4) by whether or not the evader can escape when *pursued* by the pursuer.

Since the pursuer is here presumed to prefer the evader's escape to a simultaneous kill, it could be advantageous for the pursuer to *cooperate* with the evader and ensure his escape. If so, the outer boundary of the SK region in Fig. 1 should be replaced by a lower boundary related to such cooperation. This is shown in Fig. 2 (for $H = 150$ deg, $\beta = 2$). To the right of (L), boundary (1) has been repalced by a new boundary (3), passing through the intesection of boundaries (1) and (2). The associated adjoint vectors ∇V_i must be related by:

$$\nabla V_3 = \alpha \nabla V_1 + (1-\alpha) \nabla V_2$$

where the mixing parameter α is uniquely determined by the vanishing of the Hamiltonian $\underset{P}{\mathrm{Max}}\ \underset{E}{\mathrm{Max}}\ \dot{V}_3$.

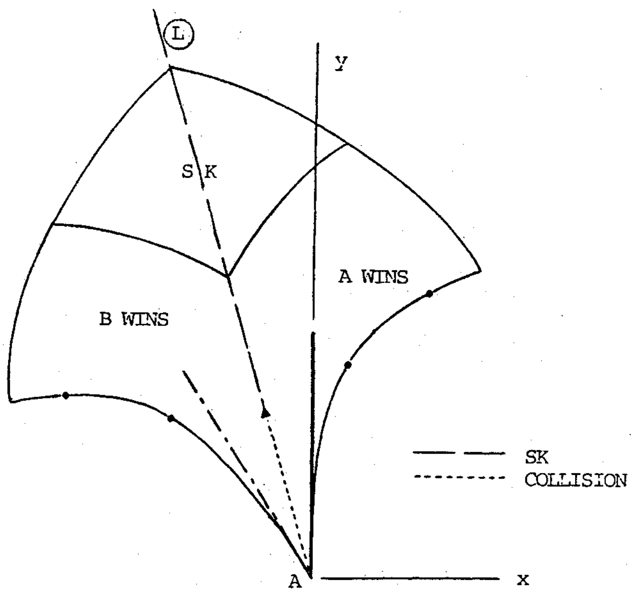

Fig. 1 Outcomes for identical aircraft ($H = 150$ deg, $\beta = 2$).

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